

$-\pi \leq x \leq \pi$ . Eq. (7) states that for a given number  $2N+1$  of Fourier terms in the expansion of  $g(x)$  the function  $f(u)$  may be modified from a  $\sin \pi u / \pi u$  form by rearranging  $2N$  of the zeroes of this latter function in an arbitrary manner. The synthesis procedure is now reduced to that of choosing a suitable polynomial  $P(u)$ . The particular case of approximating a Chebyshev behavior for  $f(u)$  is discussed by Collin<sup>2</sup> and Taylor.<sup>3</sup> Further applications to transmission line taper synthesis will be presented in a later paper. Finally, the antenna optimization problem is discussed in greater detail elsewhere.<sup>4</sup>

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<sup>4</sup> R. E. Collin and S. Rothschild, "Reactive Energy in Aperture Fields and Aperture  $Q$ ," to be presented at the Copenhagen Electromagnetic Symposium, June, 1962.

## A Proposed Lossless Electronic Phase Shifter\*

Modern antenna array designs frequently require the use of high-speed, electronically controlled phase shifters. The use of low-noise receivers now becoming available also dictates the requirement that such phase shifters have zero, or very small, losses. The use of parametric devices as active elements in such phase shifters is particularly attractive because of their high degree of reliability, high frequency capability, and excellent noise performance. This memorandum proposes such an active electronic phase shifter, which accomplishes the additional function of providing amplification or attenuation independently of its phase shift.

Although most parametric amplifiers have been designed to amplify an incoming signal directly without frequency conversion, it is possible to design low-noise parametric up-converters with the output frequency higher than the input frequency. The reverse procedure of down-converting is not practical because the power gain is always less than unity. There are two distinct modes of operation for the parametric up-converter. The output frequency may be either the sum or the difference of the pump and signal frequencies. When the lower idler is utilized ( $\omega_p = \omega_s + \omega_i$ ), the amplifier is a negative resistance device with unlimited gain and is potentially unstable. However, when the upper idler is employed ( $\omega_i = \omega_p + \omega_s$ ), the up-converter is stable but the gain is limited to the ratio of the idler frequency and signal frequency.

Changes in pump frequency appear as changes in the output frequency unless the

idler is down-converted in a conventional crystal mixer which uses the same pump source for the local oscillator. In this case the original input frequency is recovered. Although the output frequency of the parametric up-converter/down-converter combination is independent of pump frequency for fixed line lengths, the phase shift through the combination is a function of pump frequency.

It has been shown that the expression relating the phase terms in the expression for the voltages at the three different frequencies is similar to the expression relating the frequencies with an additional phase shift of  $-\pi/2$  radians due to the capacitive reactance of the varactor.<sup>1</sup> The expression for the phase shift in a crystal mixer does not contain the term  $-\pi/2$ . The phase shift in a lower sideband up-converter is given by

$$\phi_p = \phi_s + \phi_i - \frac{\pi}{2}$$

where  $\omega_p = \omega_s + \omega_i$ , while the phase shift in a crystal mixer is given by

$$\phi_p = \phi_s + \phi_i.$$

With the aid of Fig. 1 the following set of equations may be written:

$$\begin{aligned}\phi_{s_0} &= \phi_{p_2} - \phi_{i_2} \\ \phi_{p_3} &= -\frac{2\pi l_2}{\lambda_p} \\ \phi_{i_2} &= \phi_{i_1} - \frac{2\pi l_3}{\lambda_i} \\ \phi_{i_1} &= \phi_{p_1} - \phi_{s_1} - \frac{\pi}{2} \\ \phi_{p_1} &= -\frac{2\pi l_1}{\lambda_p}.\end{aligned}$$

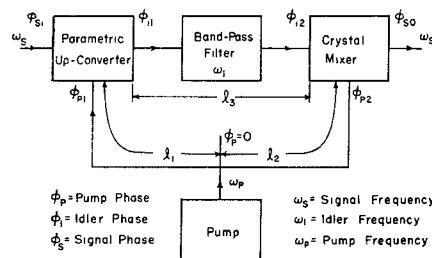


Fig. 1—Block diagram of parametric up-converter/down-converter system.

The solution for the total phase shift is

$$\begin{aligned}\phi &= \phi_{s_0} - \phi_s \\ &= \frac{2\pi}{\lambda_p} (l_1 - l_2) + \frac{\pi}{2} - \frac{2\pi l_3}{\lambda_i}.\end{aligned}$$

By similar manipulation, the phase shift of the upper sideband up-converter/down-converter combination is

$$\begin{aligned}\phi &= \phi_{s_0} - \phi_s \\ &= \frac{2\pi}{\lambda_p} (l_2 - l_1) - \frac{\pi}{2} - \frac{2\pi l_3}{\lambda_i},\end{aligned}$$

D. B. Anderson, and J. C. Aukland, "Transmission Phase Relations of Four-Frequency Parametric Devices," presented at IRE Microwave Theory and Techniques Nat'l Symp., Washington, D. C.; May 15-17, 1961.

which is the negative of the previous expression.

Thus, for either the lower sideband or the upper sideband up-converter/down-converter combination, the total phase shift at a given signal frequency is a function only of the pump frequency for fixed line lengths. The choice between an upper- or lower-sideband up-converter will be affected by gain and stability requirements.

Since the rate of change of phase shift vs pump frequency is controlled by the line lengths, it is possible to design for any specified range of phase shift, 360° for example, over a fixed range of the tunable pump source. This tuning range is determined by the bandwidth of the band-pass filter, which in turn is dictated by the signal frequency.

For example, assuming a signal frequency of 200 Mc and an  $X$ -band pump, a 200-Mc band-pass filter centered on 9800 Mc would support the lower idler frequency for pump frequencies between 9900 Mc and 10,100 Mc, while suppressing the upper idler. With this set of conditions, a difference of electrical line lengths between  $l_2$  and  $l_1 + l_3$  of 1.5 meters would be required to give 360° phase shift over the operating range.

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## Oversize Waveguide Directional Coupler\*

Oversize rectangular waveguide using  $TE_{10}$  mode propagation has been suggested as a means of reducing the attenuation and fabrication difficulties of transmission lines and components for frequencies above 40 Gc.<sup>1,2</sup> Experiments on straight sections of such transmission lines show that it is superior to pure optical transmission for short distances.<sup>3</sup>

A directional coupler using oversize rectangular waveguide has been constructed; experimental results are described in this communication. The initial device was de-

\* Received May 1, 1962.

<sup>1</sup> A. F. Harvey, "Optical techniques at microwave frequencies," *Proc. IEE*, vol. 106, pt. C, pp. 141-157; March, 1959.

<sup>2</sup> L. Lewin, "A Note on Quasi-Optical Methods at Millimeter Wavelengths," *PBI-MRI Symp. on Millimeter Waves*, Interscience Publishers, New York, N. Y., p. 469; 1959.

<sup>3</sup> R. H. Garnham, "Optical and Quasi-Optical Transmission Techniques and Component Systems for Millimeter Wavelengths," Royal Radar Establishment, Malvern, England, RRE Rept. No. 3020; March, 1958.

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